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Motion degraded bar spread functions of annular aperture systems

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The influence of linear motion and transverse sinusoidal vibrations on the images of an incoherently illuminated single bar has been studied for various sizes of the central obstruction of the pupil. It is found that degradation of the bar spread function is more in the case of transverse sinusoidal vibrations than in case of linear motion. Further, as the size of the central obstruction is increased, the deleterious effects of motion are seen to be of greater and greater significance. In fact, for hignly obscured annular apertures, the tolerances on the image motion appear to become very stringent. Results have also been obtained for the images of two other isolated extended objects, namely a triangle and an object in the shape of a single cycle of a pure sinusoid.

1. Introduction

Recently, methods of Fourier optics have been very successfully and extensively used for the study of images formed by diffraction limited and aberration limited optical systems. It has also been possible to discuss degradations of the images due to non optical factors such as image motion etc. by these methods. In this communication, we use Fourier optics methods to study the properties of the motion degraded bar spread functions.

Single bar analysis has acquired importance in the assessment of optical systems only recently. A single bar is of direct importance in aerial photography as discussed by Brock (1970a) because the projected silhoutes of such objects as pipes, rails, white road lines, highways etc., are effectively single bar objects and their imagery is of considerable importance The spatial frequency spectrum of a single bar turns out to be the familiar sine function and in the limit single bar turns out to be an infinitesimally narrow pulse whose spectrum extends at a uniform level over all frequencies. The wide spectral band width of a single bar provides an explanation for the frequent appearance in aerial photography of isolated objects whose diameter is well below the resolution limit. A single bar target has also found application as a resolving power test target (Brock

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1970b, Lewis & Hauser 1962). In particular, the methods of defining and calculating the contrast of bar images have been developed by Charman (1965), William (1969) and Sokoloskiy (1973).

In the present work, annular aperture optical systems are considered. An extensive list of references dealing with studies of images formed by such systems is given elsewhere (Singh & Dhillon 1969, Singh & Kavathekar 1969, Katti *et al* 1969). Some of the recent studies connected with the application of annular aperture systems and the properties of the images formed by them are by Mangus & Underwood (1969), Foreman, Jr. *et al* (1971), McCrickerd (1971), Powell (1973), Price & Vinter (1973) and Tschunko (1974). In many of the applications (Montgomery & Adams 1970, Cameron *et al* 1971), image motion is a very important factor and severely limits the performance of image forming systems. In some cases, techniques for image motion compensation or stabilization have been considered by Bottema *et al* (1969) and by Bottema & Woodruff (1972). Work has also been reported on the influence of motion on image quality using third order aberration theory by Bottema & Woodruff (1971) and by Weatherall & Rimmer (1972). In the present work, we present the results of our studies on the influence of linear motion and transverse sinusoidal vibrations on the diffraction images of a single bar formed by annular aperture systems. Results of similar studies in the case of two other isolated extended objects, namely a triangle and an object having the shape of a single cycle of a pure sinusoid are also included.

2. Theory

The image intensity spectrum $I(w)$ and the object intensity spectrum $O(w)$ are related by

$$I(w) = T(w).O(w), \tag{1}$$

where $T(w)$ is the optical transfer function of the system and w the normalised spatial frequency. The intensity distribution $i(v)$ in the image is clearly the inverse Fourier transform of $I(w)$ Hence using eq. (1) we get

$$i(v) = \int_{-2}^{2} T(w).O(w) \exp(ivw) \mathrm{d}w. \tag{2}$$

In this equation, v is the distance in the image plane in diffraction units. Since a one dimensional Fourier transform has been written in eq. (2), the formula is applicable only to images of one dimensional objects. The object spectrum $O(w)$ can be obtained by taking the Fourier transform of the object intensity function $O(v)$ for a single bar. This turns out to be

$$O(w) = (\pi w)^{-1} \sin(wL), \tag{3}$$

where $2L$ is the width of the bar as given by Singh *et al* (1972, 1973). Substituting eq. (3) in eq. (2) we get

$$i(v) = 2(\pi)^{-1} \int^{\infty} T(w) \sin(wL) \cos(vw)(w)^{-1} dw. \quad (4)$$

The optical transfer function of a diffraction limited system may now be multiplied by the optical transfer function of the motion to give the transfer function of a system afflicted with motion induced image degradations. Hence,

$$T(w) = D(w)C(w), \quad (5)$$

where $D(w)$ is the diffraction limited MTF and $C(w)$ the motion MTF. The expression for $D(w)$ has been quoted by Singh & Dhillon (1969) and for an annular aperture system with obstruction ratio η may be written as

$$D(w) = (1-\eta^2)^{-1}(A+B+C), \quad (6)$$

where

$$\begin{aligned}
A &= \pi^{-1}2[\cos^{-1}(w/2)-(w/2)(1-w^2/4)^{\frac{1}{2}}]; && 0 \leqslant w/2 < 1,\\
&= 0; && w/2 \geqslant 1.\\
B &= \pi^{-1}2\eta^2[\cos^{-1}(w/2\eta)-(w/2\eta)(1-w^2/4\eta^2)^{\frac{1}{2}}]; && 0 \leqslant w/2\eta < 1,\\
&= 0; && w/2\eta \geqslant 1.\\
C &= -2\eta^2; && 0 \leqslant w/2 < (1-\eta)/2,\\
&= -2\eta^2+\pi^{-1}2\eta \sin\phi+\pi^{-1}(1+\eta^2)\phi \\
&\quad -\pi^{-1}2(1-\eta^2)\tan^{-1}[(1-\eta)(1+\eta)^{-1} \tan\phi/2]; && (1-\eta)/2 \leqslant w/2 < (1+\eta)/2,\\
&= 0; && w/2 \geqslant (1+\eta)/2.
\end{aligned}$$

In the above expressions

$$\phi = \cos^{-1}[(1+\eta^2-w^2)/2\eta].$$

Further, the MTF for linear motion is given by

$$C(w) = \mathrm{sinc}(\pi A w), \quad \ldots \quad (7)$$

where

$$A = (Vte/\lambda F)$$

te being the time of exposure, v the velocity of image motion, λ the wavelength of incident radiation and F the f-number of the system. For transverse vibrations, the OTF is given by Trott (1960) as given below

$$C(w) = J_0(\pi A w), \quad \ldots \quad (8)$$

where

$$A = (2f\psi/\lambda F),$$

f being the focal length of system and ψ the maximum angular movement.

We may also mention that the object spectrum for a triangle shaped object is given by

$$O(w) = (\pi w^2)^{-1} \sin^2(wL) \qquad \cdots \quad (9)$$

and for an object in the shape of a single cycle of a pure sinusoid by

$$O(w) = \{2w(\pi^2 - 4w^2L^2)\}^{-1}.\pi\sin(2wL). \qquad \cdots \quad (10)$$

Object intensity spectrum $O(w)$ for these two isolated extended objects is given by Singh & Jain (1972) and by Singh *et al* (1973). Knowing $O(w)$, one can again write down the expressions for their motion degraded images following the procedure outlined above for a bar.

We would also like to mention that our work discusses the degradation of bar images in terms of diffraction limited optical systems. This is not only of academic interest but the OTF's of many real optical systems may be reasonably approximated by an equivalent diffraction limited OTF especially if we confine ourselves to the portion which lies above the threshold or the lower useful value of the modulation set by the detecting device e.g., the photographic emulsion in aerial photography.

Another complication arises, that is, how does one define and compute the modulation of the image since any peak value against a background of 0.0 gives a modulation of 1.0 when calculated according to conventional formalism, namely,

$$\text{Modulation} = (I_{max} - I_{min})/(I_{max} + I_{min}), \qquad (11)$$

where I_{max} is the peak value and I_{min} is the background. Williams (1969) has shown that the final modulation of a single bar target of original modulation Mi, which is degraded according to some MTF, is equal to $b.Mi$. Here b is the final peak value, in normalized intensity units, of an equal size bar image with an input intensity and modulation of 1.0 which has been degraded by the same $M.T.F.$ Therfore, the final modulation of bar images which had an input intensity and modulation 1.0 is defined to be their maximum intensity or effective exposure value, b. Considering this definition as an extension from the cases where the input modulation is less than 1.0, this definition is very clear and should not be confused with the complication mentioned above forbidding the use of eq. (11) for single bar analysis. It must be noted that this definition may be extended to define a longrithmic contrast as was done by Charman (1965). However, we characterize our images by the ordinary contrast.

3. Numerical Results and Discussions

Intensity distributions were calculated numerically on an electronic computer using a 20-point Gauss Quadrature method. Results of these calculations are shown graphically in figures 2–4 for a bar width $2L = 2.0$. In some cases, the results in the absence of image motion ($A = 0.0$) are also shown for the sake of comparison. Figure 1 demonstrates the influence of linear image motion on

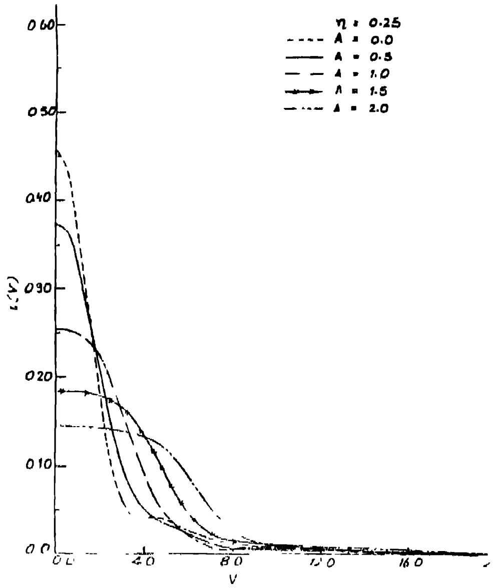

Fig. 1(a) Irradiance distribution in the image of a bar formed by an annular aperture of obscuration ratio $\eta = 0.25$ and for linear image motion parameter $A = 0.0$, 0.5, 1.0, 1.5, 2.0.

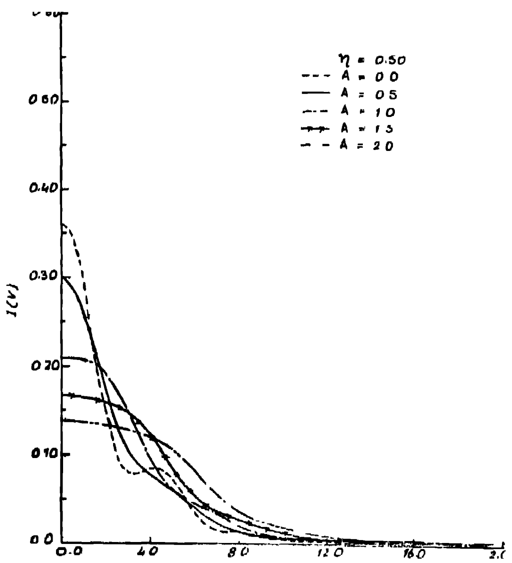

Fig. 1(b) Same as figure 1(a), $\eta = 0.50$.

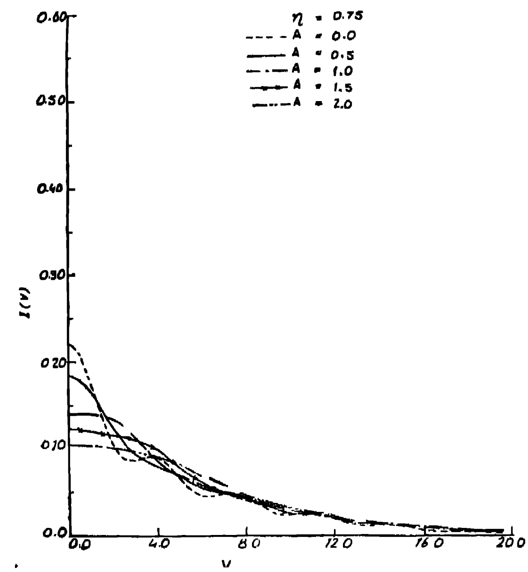

Fig. 1(c) Same as figure 1(a), $\eta = 0.75$.

the bar images formed by annular aperture systems with obstruction ratio $\eta = 0.25$, 0.50 and 0.75. In all the three cases, it is observed that the secondary maxima in the motion free images arising due to diffraction at the annular aperture are no longer visible in the motion degraded images. Also as the value of η increases, the degradations due to motion are more deleterious than

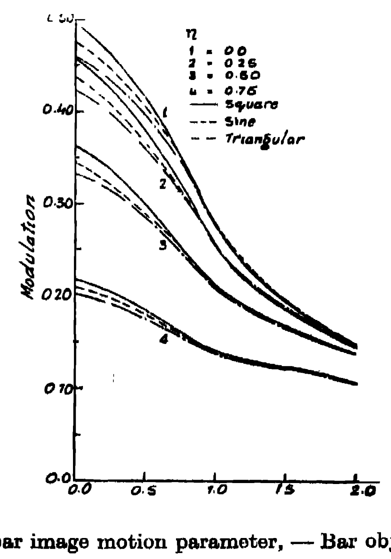

Fig. 2. Contrast vs linear image motion parameter, — Bar object, - - - object of the shape of a single cycle of a sinusoid, — - — - object of the shape of a triangle.

for small value of η. This clearly means that comparatively smaller amounts of motion are tolerable in cases of systems with high obstruction ratios. In figure 2, we have plotted the modulation of the final image as a function of the image motion parameters for different obstruction ratios when the object is in

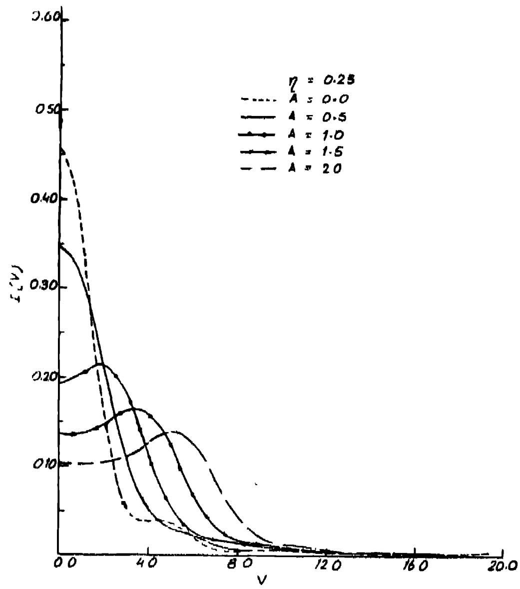

Fig. 3(a) Same as figure 1(a), transverse sinusoidal vibration. parameter $A = 0.0,\ 0.5,\ 1.0,\ 1.5,\ 2.0$; $\eta = 0.25$.

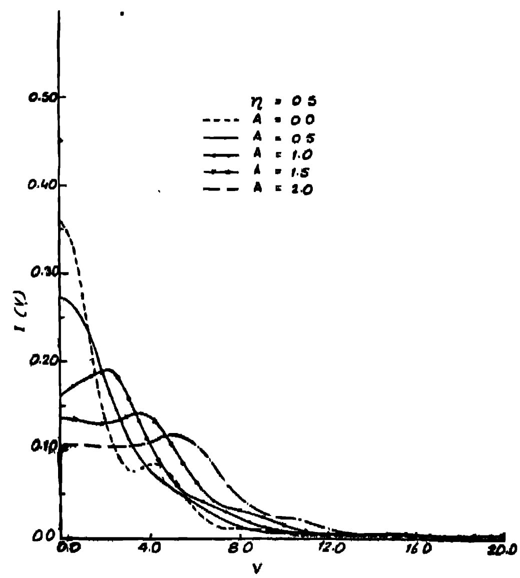

Fig. 3(b) Same as figure 3(a), $\eta = 0.50$.

the form of a single cycle of pure sinusoid, a triangle and a bar. Conclusions similar to those derived from figure 1 are applicable in these cases also. It must also be observed that the differences in the images of these types of isolated extended but aperoidic objects are not considerable.

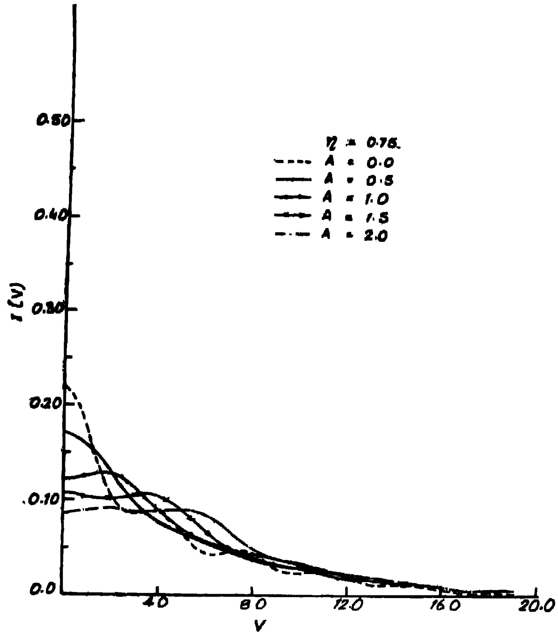

Fig. 3(c) Same as figure 3(a), $\eta = 0.75$.

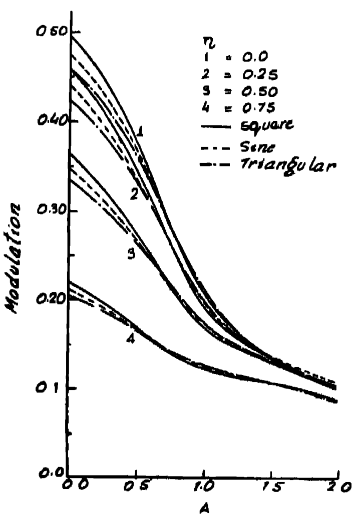

Fig. 4. Same as figure 2 but in case of transverse sinusoidal vibrations.

The case of transverse sinusoidal vibrations is then considered and the results of calculations shown graphically in figure 3 to demonstrate the properties of vibration degraded images of a bar. The general trends of the results are similar to those of figure 1. In this case, the degradations are more pronounced with the appearance of images showing phase reversals as soon as A takes up some value between 0.5 and 1.0. Once again, the suppression of secondary characteristic of annular aperture diffraction due to the presence of transverse sinusoidal vibrations should be interestingly noted. Figure 4 is similar to figure 2 and shows that vibrations and linear motion lead to similar types of degradations.

Finally, a comparison of figure 2 with figure 4 reveals that transverse sinusoidal vibrations are more deleterious than linear image motion.

4. Concluding Remarks

It is well known that the transfer function of certain optical systems with appropriate amplitude filters is very similar to the transfer function of annular aperture systems as shown by Katti *et al* (1970). Hence, the trends for motion degraded bar spread functions in these will be similar to our results. Further, since the tolerances on image motion turn out to be very stringent for higher obstruction ratios, more sophisticated and efficient image motion compensation and stabilization techniques must be developed and used to obtain the full benefit of annular aperture diffraction.

We would also like to mention that the vibration and linear motion degraded images of two dimensional objects such as disks, annulii etc. cannot be calculated using eq. (5) for the OTF of the total system because in these cases $C(w)$ does not possess a rotational symmetry but acquires a certain azimuthal dependence. Work is in progress in this direction also.

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